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AUTHOR Munro, John
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ABSTRACT

This paper examines a model of individual ways of learning and its implications for mathematics teaching. Topics discussed include: alternative ways that students use to represent mathematical ideas, management or control mechanisms, related models of learning preferences, ways in which students relate and manipulate ideas, and implications of this model of individual learning preferences for the teaching of mathematics. Contains 10 references. (MKR)

Multiple intelligences and mathematics teaching

John Munro

Curriculum, Teaching and Learning,

University of Melbourne,

Parkville, 3052

Telephone number : (03) 3448230

Fax : (03) 347 2468

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Multiple intelligences and mathematics teaching

John Munro

It is generally recognized that mathematics ideas are learnt via constructive or building processes (von Glasserfield, 1991). Differences in the ways in which students do this have received less attention. In particular, the influence of preferred ways of learning on the construction of mathematics ideas and the implications of these for how we teach mathematics have attracted little interest. This paper examines a model of individual ways of learning and its implications for mathematics learning.

We are all familiar with the notion that people learn mathematics in different ways. Some people remember best what they have seen. Others are good with words. Some may be competent in solving problems but have difficulty learning mathematics formulae. You may have students in your class who are very good with their hands or who have a creative, artistic talent and flair but who have difficulty with more formal mathematics learning and who do not see themselves as able learners at all. If we are to meet the learning needs of these students and to provide a more inclusive mathematics curriculum, we need to examine ways of accommodating these individual ways of learning in our teaching. We also may be interested in assisting our students to understand and value their own preferred ways of learning and to broaden the ways in which they go about learning mathematics.

A model of the learning process Learning involves a change or re-organization of an individual's knowledge base in some way. As a first approximation to explain how this occurs, a three-stage process is proposed; students need to (1) attend to the information that will lead to the re-organization, (2) activate processes that lead to the organization and (3) demonstrate in some way that the change or reorganization has occurred. Students differ in how they attend to particular information, how they think about it and how they show what they have learnt. This model is shown diagrammatically in Figure 1.

Incoming information used for learning	→	The learner builds a new idea by re-organizing or adding to what is already known	→	The learner displays the new idea in some way

Figure 1 : Proposed model of learning

This model of learning is deceptively simplistic and ignores several processes implicated in learning. We know, for example, that what we see or hear is determined in part by what we expect to see or hear; our existing knowledge at any time influences what we perceive. We know also that we are more likely to invest our attention in incoming information that challenges us in some way or that helps us to solve problems that confront us at the time. As well, the distinction between the perceiving and the 'thinking' processes is not clear-cut; while we are thinking about an idea we may frequently 'return' to attending to the information, looking for something that we didn't notice earlier; there is not a 'one-directional' flow of information from input to thinking to output. We can also attend simultaneously to several sources of information at once. When we read a number sentence, for example, we can attend at once to the whole number sentence, the context in which it used and individual number information. We make decisions using whatever information we believe helps us most. Bearing in mind these limitations, this model provides a first approximation to understanding learning. What we are looking for are dimensions on which students differ when they learn, leading to individual ways of learning.

Preferred ways of thinking about the ideas being learnt .

A major dimension for looking at differences in how people learn mathematical ideas is how they go about 'representing' or 'coding' them. It is proposed (Munro 1993 a) that students have access to several alternative ways for representing or coding mathematical ideas :

(1) **verbal/linguistic representation**; knowing by making use of one's understanding of words and the properties of language, that is, thinking by using words, sentences and verbal propositions. This is used when students convert numerical data to a verbal code, tell themselves what number statements are saying or what a particular diagram is showing. When students talk to themselves or to others about mathematical ideas, they can use their verbal logic and reasoning more easily. People using this way of representing ideas learn and think most easily by discussing, arguing, debating and following spoken explanations. They think in words rather than pictures, often need to talk to themselves as they are learning, listen to themselves and listen to other people. These students frequently benefit from 'process approaches to mathematics learning'. They remember best information that they have converted to a verbal code. Students who represent ideas in this way may have difficulty using their knowledge to solve real-life problems. Also, they may need time to translate their ideas into actions.

(2) **logical/mathematical representation**; understanding by using abstract concepts and symbols such as mathematical symbols, scientific symbols and by using 'scientific logic' or reasoning to link ideas. Students build ideas using this way of coding by reasoning inductively and deductively. They look for logic, order and consistency, the ways in which the ideas are organized or related, for example, cause and effect. They analyse patterns, make objective observations, draw conclusions and formulate hypotheses as well as applying general rules to particular situations. They find it easy to understand and to use mathematical formulae. They prefer things to be organized and logical.and may be said to have a 'scientific mind'.

(3) **visual/spatial representation**; understanding by making nonverbal images of the ideas to be learnt. Often these images seem to have a visual quality; information is stored in visual codes, noting spatial relationships, patterns and properties. This representational form allows students to relate ideas in terms of their spatial or temporal proximity with other ideas, rather than in terms of logical or linguistic relationships. Ideas that occur in the same event or episode are often assumed to be related. Students using this format often draw pictures of what they are learning while they are learning. The student who visualizes 23 as two bundles of ten sticks and three loose sticks is using this form of representation. This type of knowing often allows students to think in terms of wholes; specific pieces of information are 'slotted' into a mental picture; the whole picture provides a set of 'visual hooks' on which particular aspects can be hung. Often the items can be related spatially and students can manipulate them by moving them around, rotating them in the picture, etc. They can act on the mental picture in logical ways, for example, imagining three and one half pizzas being re-arranged so that each whole is cut into halves.

(4) **body/kinaesthetic representation**; understanding by using actions to represent ideas being learnt; ideas are represented by characteristic actions that may have a directional component. For many children, doing physical actions first can lead to learning mental actions and operations. Mathematical ideas are represented in this way through actions. The idea that eighty seven is equal to seventy and seventeen can be understood by changing one of the tens to ten ones. The focus here is not on whether the action is applied to a quantity of sticks or to a number but rather on the re-arranging action. Pupils who use this format often need to learn 'with their bodies', for example, when learning about graphs they might move their finger to map out a straight line, parabola or circle. They can remember ideas well when they use characteristic actions or gestures that stand for the ideas, for example, in tests. Pupils who prefer to use this format often seem to learn well with their hands and may find it hard to remember the names of things. When they need to show what they know in words or symbols they will probably need time to translate their action knowledge into the alternative forms.

(5) **rhythmic representation**; knowing by using rhythm, repetitive patterns and rhyme. Learning ideas by rote or by chanting, for example, the six times table 'by heart' exemplify this representational format. What is learnt is a whole image or a complete episode; the component parts can only be retrieved by taking apart the complete episode and looking for logical patterns (where these exist) between the items. Students use this format when they recall an idea by embedding it in a rhythm.

(6) **affective / mood representation;** knowing in terms of affect, emotional feeling, or mood. This way of knowing involves students recognizing mathematical ideas as interesting, challenging, boring, frustrating, etc. Learning a new idea requires motivation and persistence by learners and a level of self-confidence. Students differ in their level of self-motivation for learning an idea and how they can be motivated. Each student will have already linked a level of

motivation for learning and interest in a particular idea; whether the student values achieving the outcome and has done similar tasks in the past with positive outcomes. Students also differ in how they have attributed success and failure in mathematics in their past and this can influence current learning. Less successful students are more likely to attribute success to luck or to external sources and failure to their own stable learning capacities. Level of persistence on tasks can also influence learning. Some students display high levels of persistence for most mathematics tasks while others show fluctuating levels of persistence. Learning effective self-talk is useful here.

(7) **interpersonal representation**; knowing in terms of historical, social, cultural or religious knowledge. These areas for an individual consist of an amalgam of verbal and nonverbal imagery knowledge integrated into a network of propositions. Students from different cultures, for example, when exposed to the same mathematics teaching, may attempt to learn it in different ways. Learning from within a cultural perspective that encourages unquestioning construction of the ideas as accurately as possible may lead to a different outcome from learning within a perspective that encourages questioning and successive approximations. Learning from a perspective that sees no gender difference in access to mathematics learning will be different from a perspective that believes that males have a greater right to mathematics learning than females.

These representations can be seen as ' mental garden beds' in which ideas can be developed.

A management / control mechanism Learners develop a management or control mechanism by which they can switch between alternative ways of representing the idea. When they are finding it hard to build an idea using one way of knowing, for example, finding it hard to read a mathematics problem by verbalizing the ideas, they can change to a visualizing mode or a logico-mathematical mode. This ability to direct and regulate one's learning, to evaluate its effectiveness in terms of some goal or purpose and take further strategic action if necessary is seen as an aspect of metacognition (Cross & Paris, 1988). People gradually develop a knowledge of how they learn in mathematics; an awareness of how they feel about these ideas and how they prefer to learn them. This mechanism similar to the central executive component of short-term working memory (Baddeley, 1986, 1990; Daneman, 1987).

One aspect of this management is the extent to which students prefer to take responsibility for their learning and to manage and direct their own learning. Some show a high level of dependence and need to be structured when learning mathematics. They usually feel more secure in externally structured contexts and learn better from directing, more authoritative teachers than from more permissive, democratic teachers. Others are more independent and display more initiative.

Related models of learning preferences This proposed model of learning preferences in mathematics is derived from Paivio's dual coding theory (Clark & Paivio, 1991) and from Gardner's (1985) multiple intelligence model. The dual coding theory proposes that people can represent ideas in at least two ways; either by using their verbal knowledge (individual word meanings, relationships between meanings, verbal propositions, knowledge of grammar, pragmatics) or by using nonverbal imagery (images for concrete and abstract concepts, actions, emotions and perceptual information, etc). The fractional numeral 3/4 could be thought of in terms of a word statement such as "three out of four equal parts" or in terms of spatial imagery such as a circle divided into four equal parts with three shaded or as a pizza cut into quarters with three parts on a plate. The imagery may be stored in terms of particular episodes or events.

Although students can use both systems they differ in their capacity to do so. Some use imagery spontaneously across a range of situations, while others find it harder to use and invest attention in doing so (Clark & Paivio, 1991). These people are more able to use their verbal knowledge.

Gardner's (1985) multiple intelligence theory proposed that there are at least seven ways of 'knowing' ideas; verbal/linguistic (thinking in words and one's knowledge of language), logical/mathematical (thinking in symbolism, scientific logic and order), visual/spatial (coding in visual imagery and using spatial information to relate parts of the image), body/kinaesthetic (coding ideas in actions), musical/rhythmic (thinking in musical or rhythmic patterns), interpersonal (knowing from other people's perspectives, understanding other people's moods and feelings) and intrapersonal (knowing about one's self as a learner). Individuals show a preference for some ways of learning over others.

While Gardner's model provides an insight into alternative ways of knowing, it leaves unanswered several questions (Munro 1993a). One relates to the interplay between the different ways of knowing. Are learners restricted to using one format for representing an idea or can they draw on two or more ways at once ? Intuitively students seem to be able to draw on two or more ways of coding the idea at once, often with one way dominating. Most ideas learnt in mathematics seem to have an affective component..

A second question relates to why a learner may show a preference for a particular cluster of ways of knowing. This preference can be explained in terms of how learners allocate attention when learning. Learners need to invest attention in an idea to learn it. The amount of attention available for learning is limited. Some of this may need to be invested in using a particular format. The more a learner uses a particular format the more automatically it is used and the less demand it makes on attentional resources. Learners prefer formats that demand least attention.

In summary then, the proposed theory of learning preferences (Munro 1993a) suggests that people use a range of ways of representing or coding mathematical ideas. It differs from Gardner's theory in the following ways; it

(1) proposes a management mechanism,

(2) introduces an affective / mood way of knowing to account for the influence of beliefs and attitudes on mathematics learning,

(3) recognizes the influence of culture, history and religion on learning mathematical ideas,

(4) broadens the body / kinaesthetic way of knowing to focus on mental actions and

(5) broadens the musical / rhythmic representation to link it with learning by using rhythm or rote.

Students differ in their preference for learning by themselves rather than by working with others. It is often useful to think of a continuum here, ranging from the internally motivated self-driven learner who usually prefers to learn independently to the learner who prefers to be externally motivated learner who is driven by others in the learning situation. Some students prefer to be directed or structured in most learning situations. They are reluctant to take large risks by themselves. Others prefer to structure and to organize themselves. They prefer to direct and to manage their own learning. They dislike being directed by others and like to take control of their won learning as quickly as possible.

Relating the ideas represented

As well as alternative ways of representing or coding mathematical ideas, students differ in how they work on the ideas, that is, how they manipulate them.

Analytic or holistic strategies ? In mathematics learning ideas can be manipulated in two ways (Munro 1993a). They can be analysed into parts that are then linked up in various ways to build an overall idea. Alternatively, they can be integrated with other ideas, with each idea being treated as a whole rather than being analysed into parts. The first type of strategy is described as serial or analytic while the second is global or holistic. Students using holistic strategies are concerned with the general idea and tend to experience events globally. They need to have the whole idea explained before they can make sense of the parts. They need to learn how to move from the general to the particular and often find it difficult to note or to remember the details of the idea. These strategies are not useful when students have to learn ideas that require analysis or segmentation. Students using serial or analytic strategies work on a piece of information at a time and gradually build an overall understanding of the idea in a step by step way. These strategies allow students to segment the whole into parts and to work on the parts first. The students gradually fit the bits together to form the total picture from bits at a time.

Intuitively one would expect that these two types of learning strategies can be applied to each of the representation formats. Ideas represented verbally, for example, can be analysed in terms of components of meaning or other language properties. They can also be integrated in various ways.

While most learners use holistic or analytic strategies selectively when learning mathematics, some use one excessively. Holistic learners may tend to ignore or miss specific details. Serial learners, on the other hand, have difficulty 'getting above' the detail level and 'seeing big idea'. Holistic learners are more likely to be flexible in their thinking and prefer more open-ended learning situations; these students can often tolerate ambiguity and unanswered questions. Serial learners prefer less flexibility, convergent learning tasks and more highly structured learning situations. They tend to reflect and think about an idea often for a long time, while holistic learners are more likely to jump in and 'guestimate'.

Implications of this model of individual learning preferences for the teaching of mathematics

Present mathematics ideas in a range of ways. When teachers are teaching an idea, they can vary how they present it; in words either visually or auditorily, in pictures or as a series of actions. These things are under the control of teachers; they can select the input format.

Cueing students to think about the idea in different ways Teachers can't control how students think about the ideas being learnt. Even if ideas are presented visually, this does not mean that the students will visualize images. Shown a picture of a quadrilateral, some students may visualize it, some may tell themselves about it, some may think by acting on the sides or angles and some may think of the logical relationships shown. Teachers can suggest to students to how they might think about it, for example, to remind them to visualize or to verbalize it to themselves. Reminding students to 'make a picture' as they read a number sentence is more likely to lead to a visual image of the sentence while reminding them to verbalize as they read is more likely to stimulate the use of the verbal coding system. Use of any representational format can be cued by the teacher. Students can learn to cue themselves to think about how they are learning.

Switching ways of thinking about ideas. Building ideas in one representational format doesn't suit all students. Many students can be helped to learn an idea when they are cued to think of it in different ways. A year ten student recently had difficulty thinking about simultaneous linear equations. When the student drew a large set of rectilinear axes in the playground, drew on this the locus of the two lines and then had peers walk along each line, the student could see how both lines had one point in common. This student had found it hard to understand the problem simply by looking at pictures of the graph. When encouraged to imagine acting on the picture in this way he found he could form an impression of what it was about. When this student was encouraged to reflect on the situations in which he could learn mathematics, 2 September, 1993.

Ms Mary Rice,
FAculty of Education,
Deakin University
Waurn Ponds.

Dear Mary,

Please find enclosed a hard and disc copy of my paper for the MAV Conference Handbook. If there are any queries I can be contacted on 818-4619 (home) or 3448-230 (work). Thank you for allowing me the additional time because of the 'flu.

Best wishes,

John Munro

he could see that thinking about things as actions helped him. Rather than thinking he couldn't learn the idea, he learnt that switching ways of thinking can help.

Teachers can help students to learn to switch between ways of representing ideas, particularly when they find that one format is not the most useful for a particular situation. A key thing that distinguishes learning preferences is how the person transfers ideas learnt. Verbal-preferring learners when learning an action sequence may need to talk to themselves in order to learn it and may have difficulty generalizing action sequences from one context to another. When needing to handle visual-spatial information, they may talk to themselves about it, using verbal labelling. Visual learners, when required to process verbal information and symbolic information, make pictures and images in their minds.

Encouraging students to monitor how they learn best and to understand their preferred ways of learning Students can learn to understand and value their own approach to mathematics learning, to see that they can learn successfully, to understand the conditions under which they learn best and to broaden their approach to learning. From the self-valuing they can learn to value others. Teachers can encourage students to reflect on how they believe they go about learning mathematical ideas and to judge what works for them. Teachers can help them to understand that although students learn in different ways they can still be equally effective as learners and can learn the same ideas.

Students can broaden their preferred ways of learning mathematics. Teachers can encourage them to share their preferred ways of learning with peers, to share strategies and to try out alternative ways of solving problems for themselves. They can be encouraged to take risks and to experiment with additional ways of learning mathematics. Making them aware of their learning preference is empowering; it gives them a base from which they can develop further.

Teachers can encourage students to discuss how they go about thinking about mathematical ideas and give them options for doing this, for example, "Did you make a picture / talk about it to yourself ? " etc. Teachers can also cue students to think about the ideas that they are learning in particular ways and have them note whether they find it easier to think under the cued conditions. Studies by Munro (1992, 1993b) have shown this to be an effective learning activity. In one investigation secondary mathematics students were cued to represent mentally symbolic mathematics statements such as $2x + 3 = 19$ in different ways; in terms of visual imagery (students learnt to picture mentally the equation as a situation involving two bags of bolts and three more bolts having 19 bolts altogether with the task of working out the number in each bag), in terms of a verbal code (students learnt to tell themselves what the equation said and to listen to themselves as they said it; the equation was verbalized as "two times a certain number add three is equal to nineteen. I try to say it the way I talk". and in terms of actions(students were cued to think of the actions that the equation said, for example, 'First I begin with a certain number, then

I multiply it by two and add three. I end up with 19. What is the number ?"). Students selected their own preferred way of representing the mathematical statement and used it with substantial success.

Teachers can provide students with the opportunity to explore and understand their approach to mathematics learning. Over several lessons students can be presented with the same ideas in different formats (written or spoken verbal descriptions, pictorial form or acted out), note the conditions under which individual students learn best and self-evaluate. They can be helped to understand the various influences on learning and how to manipulate these when learning is not successful. Affective influences on learning can be investigated by identifying the situations in which they are more likely to be motivated or persistent. They can record in a diary their optimal learning conditions over several weeks.

Helping students to understand their strengths as mathematics learners. Students can explore strategies that match their learning preference at any time. Visual / spatial preferers can remind themselves to use mental picturing whenever they read or listen to mathematical ideas. They recall ideas by imagining what they look like and by using visual mnemonics. They organize key ideas by using pictures or schematic maps, etc. They can "print " in their minds" ideas heard.

Students who prefer to use verbal formats for learning mathematics can tell themselves what number sentences say, can put ideas 'in their own words', repeat aloud things that they hear, explain things that they are learning to themselves, listen for key ideas and link other ideas to these. They can remember things by thinking about what they heard.

Students who have a kinaesthetic preference for learning mathematics can use solid or pictorial models wherever possible and act on these, by moving parts around, talking to themselves as they do so. When these students are remembering ideas they can think of the actions that they did or do the actions with their hands. When they hear things they can focus on the actions that are being done and try to anticipate the outcomes of actions, for example, "What would this look like after I've acted on it?" The author recently worked with a Year 9 student who needed to learn Pythagorus' Theorem. Prior to a test the student learnt to run his finger around the three sides of several right-angled triangles and to say and explain the theorem while he did so. In the test on this topic he again ran his finger around the triangles given in the tasks and found that this substantially helped him to remember the theorem.

Students who have a holistic learning preference when, learning a new mathematics idea, can be aware that they will try to get a general impression of it first and then begin to fit in the smaller parts. When learning a new procedure they can first note what the procedure does and the types of tasks that it solves and then work through it again, noting specific details and then asking themselves guiding questions such as "What will I say first, second, etc...?" When remembering an idea they can think about the general context first and then give attention to noting specific details.

Students who have an analytic preference can break mathematical ideas to be learnt into small parts and work on each part at a time. They can pick out the first part, then the second part, etc. They can remind themselves to select each part at a time, for example, in, a procedure and then integrate them. They can learn to ask themselves "How do these fit together ? What is the key idea ?" The focus here is on how individual ideas are connected to each other and the students build them into a mental picture.

The relationship between teaching styles and preferred ways of learning mathematics. Mis-matches between teaching styles and learning preferences can lead to difficulty learning mathematics and a high level of frustration and anxiety. Teachers can explore the connection between their learning styles and teaching styles and the learning styles of students who learn most or least easily with them. They can use this data base to investigate ways in which they can broaden their teaching styles for mathematics. Students can also be helped to understand mis-matches between teaching and learning styles and to explore ways of managing these mismatches constructively.

Give students a range of ways of showing what they know about mathematical ideas. Many students find it hard to display their mathematical knowledge in words or in mathematical symbolism. Students who have a visual preference for learning can record ideas in drawing pictures first and then convert them to symbols or words later. Students who prefer to think linguistically can talk to themselves about ideas before they write them in symbols. Students who prefer to think kinaesthetically can act out the ideas before they write or speak about them. Some action learners try to avoid being seen to do actions. They need to be encouraged that it is acceptable and that it will help them to learn.

Help students to monitor the affective and emotional aspects of mathematics learning. Learning a new idea requires an investment of motivation and persistence from learners and a level of self-confidence. Some students believe that if they are not interested initially in a task, then they will never be interested in it and cannot be motivated to learn it. They need to experience their level of interest changing as they become more familiar with the task. Interest in a task is determined in part by whether the student values achieving the outcome and has done similar tasks earlier with positive outcomes. Teachers can set up situations in which students can investigate the effect of persistence, for example, by helping students to monitor progress towards goals and to see themselves making getting closer to their goals are important here. Learning effective self-talk statements are useful here. Students can examine the effect of changing how they attribute the success and failure in mathematics learning.

Conclusion

Every student has a unique set of mathematics learning preferences. By taking account of these teachers can ensure that their teaching is more inclusive. Helping students to understand and to value the uniqueness of their own approaches to learning is empowering; it gives them a base or starting point from which they can develop further.

There are many issues that have not been developed in this paper. Procedures for evaluating students' learning preferences, developmental changes in mathematics learning preferences, the extent to which it is influenced by the teaching environment, the relationship between learning preferences and teaching styles may be expected to attract increasing interest in the future.

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